# A Finite Element Subproblem Method for Position Change Conductor Systems

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**Abstract— Analyses of magnetic circuits with position changes of both massive and stranded conductors are performed via a finite element subproblem method. A complete problem is split into subproblems associated with each conductor and the magnetic regions. Each complete solution is then expressed as the sum of subproblem solutions supported by different meshes. The subproblem procedure simplifies both meshing and solving processes, with no need of remeshing, and accurately quantifies the effect of the position changes of conductors on both local fields, e.g. skin and proximity effects, and global quantities, e.g. inductances and forces. Applications covering parameterized analyses on conductor positions to moving conductor systems benefit from the developed approach.**

## I. INTRODUCTION

A subproblem method (SPM) with finite element (FE) solutions provides clear advantages in repetitive analyses and helps improving the solution accuracy [1]-[6]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each subproblem.

A FE-SPM is herein developed for coupling solutions of position change conductors in magnetic systems, with the aim to accurately calculate the changes of both local fields (skin and proximity effects, reaction fields, local forces) and global quantities (currents, voltages, inductances, Joule losses, forces). Both massive and stranded conductors are considered, in parameterized analyses on their positions, naturally extended to moving conductor systems.

The SPM combines any changes via volume sources (VSs), originated from previous solutions and applied via mesh-to-mesh projections. The developments are performed for the magnetic vector potential FE magnetodynamic formulation, paying special attention to the proper discretization of the constraints involved in each SP and to the resulting weak FE formulations and circuit relations. The method will be illustrated and validated on test problems.

## II. COUPLED MAGNETIC SUBPROBLEMS

### A. Sequence of Subproblems

Complete models are proposed to be split into sequences of SPs, gathering sets of conductors and magnetic regions. The SP solutions are to be added to give the complete solution. This offers a way to perform parameterized analyses, with a direct access to each change. The parameters can be the positions of the conductors, as well as their conductivities.

Each SP is defined in its own domain. At the discrete level, this aims to decrease the problem complexity and to allow distinct meshes with suitable refinements and possible domain overlapping. No remeshing is necessary when adding a region or changing its position.

### B. Canonical magnetic problem

A canonical magnetodynamic problem $p$, to be solved at step $p$ of the SPM, is defined in a domain $\Omega_p$, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. The eddy current conducting part of $\Omega_p$ is denoted $\Omega_{c,p}$ and the non-conducting one $\Omega_{c,p}{}^C$, with $\Omega_p = \Omega_{c,p} \cup \Omega_{c,p}{}^C$. Massive conductors belong to $\Omega_{c,p}$, whereas stranded conductors belong to $\Omega_{c,p}{}^C$. The equations and material relations of problem $p$ are

$$\operatorname{curl} \boldsymbol{h}_p = \boldsymbol{j}_p \,, \quad \operatorname{div} \boldsymbol{b}_p = 0 \,, \quad \operatorname{curl} \boldsymbol{e}_p = -\partial_t \boldsymbol{b}_p \,, \quad \text{(1a-b-c)}$$
$$\boldsymbol{h}_p = \mu_p^{-1} \boldsymbol{b}_p + \boldsymbol{h}_{s,p} \,, \quad \boldsymbol{j}_p = \sigma_p \boldsymbol{e}_p + \boldsymbol{j}_{s,p} \,, \quad \text{(2a-b)}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{e}_p$ is the electric field, $\boldsymbol{j}_p$ is the electric current density, $\mu_p$ is the magnetic permeability, $\sigma_p$ is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to $\Omega_p$. Note that (1c) is only defined in $\Omega_{c,p}$ (as well as $\boldsymbol{e}_p$), whereas it is reduced to the form (1b) in $\Omega_{c,p}{}^C$. Boundary conditions (BCs) on $\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}}$, $\boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}}$ or $\boldsymbol{n} \times \boldsymbol{e}_p|_{\Gamma_{e,p} \subset \Gamma_{b,p}}$ have to be defined, acting as surface sources (SSs) possibly expressed from previous solutions.

The fields $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$ in (2a-b) are VSs. The source $\boldsymbol{h}_{s,p}$ is usually used for fixing a remnant induction. The source $\boldsymbol{j}_{s,p}$ fixes the current density in inductors. With the SPM, $\boldsymbol{h}_{s,p}$ is also used for expressing changes of permeability and $\boldsymbol{j}_{s,p}$ for changes of conductivity, or for adding portions of inductors [4]-[6]. For changes in a region, from $\mu_q$ and $\sigma_q$ for problem $q$ to $\mu_p$ and $\sigma_p$ for problem $p$, the associated VSs $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$ are

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1} - \mu_q^{-1}) \boldsymbol{b}_q \,, \quad \boldsymbol{j}_{s,p} = (\sigma_p - \sigma_q) \boldsymbol{e}_q \,. \quad \text{(3a-b)}$$

Each problem $p$ is constrained via the so defined VSs and SSs from parts of the solutions of other problems. This offers a wide variety of changes [2]-[6].

Equations (1b-c) are fulfilled via the definition of a magnetic vector potential $\boldsymbol{a}_p$ and an electric scalar potential $v_p$,

$$\operatorname{curl} \boldsymbol{a}_p = \boldsymbol{b}_p \,, \quad \boldsymbol{e}_p = -\partial_t \boldsymbol{a}_p - \operatorname{grad} v_p = -\partial_t \boldsymbol{a}_p - \boldsymbol{u}_p \,. \quad \text{(4a-b)}$$

The weak $\boldsymbol{a}_p$-formulation of problem $p$ is obtained from the weak form of the Ampère equation (1a), i.e. [4],

$$(\mu_p^{-1} \operatorname{curl} \boldsymbol{a}_p, \operatorname{curl} \boldsymbol{a}')_{\Omega_p} + (\boldsymbol{h}_{s,p}, \operatorname{curl} \boldsymbol{a}')_{\Omega_p} - (\boldsymbol{j}_{s,p}, \boldsymbol{a}')_{\Omega_p}$$
$$+ (\sigma_p \partial_t \boldsymbol{a}_p, \boldsymbol{a}')_{\Omega_{c,p}} + (\sigma_p \boldsymbol{u}_p, \boldsymbol{a}')_{\Omega_{c,p}} = 0 \,, \quad \forall \boldsymbol{a}' \in F_p^1(\Omega_p) \,, \quad \text{(5)}$$

where $F_p^1(\Omega_p)$ is a curl-conform function space defined on $\Omega_p$, gauged in $\Omega_{c,p}{}^C$, and containing the basis functions for $\boldsymbol{a}_p$ as well as for the test function $\boldsymbol{a}'$ (at the discrete level, this

space is defined by edge FEs; the gauge is based on the tree-co-tree technique); $(\cdot,\cdot)_\Omega$ denotes a volume integral in $\Omega$ of the product of its vector field arguments.

## III. CONDUCTORS IN PROBLEM SPLITTINGS

### A. Adding or Changing a Massive Conductor

The circuit relation of a massive conductor $\Omega_{c,p}$, relating its current $I_p$ and voltage $U_p$ (circulation of $-\boldsymbol{u}_p$ along the conductor) changes, due to contributions from a problem $q$, is

$$(\sigma_p\,\partial_t\,\boldsymbol{a}_p,\boldsymbol{u}')_{\Omega_{c,p}} + (\sigma_p\,\boldsymbol{u}_p,\boldsymbol{u}')_{\Omega_{c,p}} - (\boldsymbol{j}_{s,p},\boldsymbol{u}')_{\Omega_{c,p}} = I_p\,. \quad (6)$$

If no current change is allowed ($I_p=0$) and proximity effects due to solution $q$ are neglected (possibly in a first step), $\boldsymbol{a}_p=0$ and (6) simply leads to a voltage change, with $(\boldsymbol{u}_p,\boldsymbol{u}')_{\Omega_{c,p}} = -(\partial_t\,\boldsymbol{a}_q,\boldsymbol{u}')_{\Omega_{c,p}}$.

For considering proximity effects, $\boldsymbol{a}_p$ and $\boldsymbol{u}_p$ need to be solved with (5) and (6), usually with $\boldsymbol{h}_{s,p}=0$ and $\boldsymbol{j}_{s,p}=\sigma_p\boldsymbol{e}_q$. This leads to the actual circuit relation change.

To illustrate and validate the SPM, TEAM problems 17 and 28 will be studied, dealing with a jumping ring and a conducting plate in levitation, respectively. These problems will be shown to be well adapted to the SPM, allowing tests of progressive levels of difficulty, from magnetostatic to magnetodynamic problems, from frequency to time domain, from axisymmetric to 3-D models, from current to voltage sources, etc., also with moving bodies. Furthermore they need accurate calculations of global quantities, e.g. self and mutual inductances and forces.

An example of result for TEAM problem 17 is shown in Fig. 1, showing the height of a conducting ring versus the input current (50 Hz) calculated with the SPM, decoupling the meshes of the magnetic source (coil and magnetic core) and of the moving ring, and the classical approach with remeshing for any new position of the ring. For a similar accuracy, a speed-up factor of about 100 is obtained with the SPM, thanks to the no remeshing and the reduction of the computational domain for each position change of the ring.
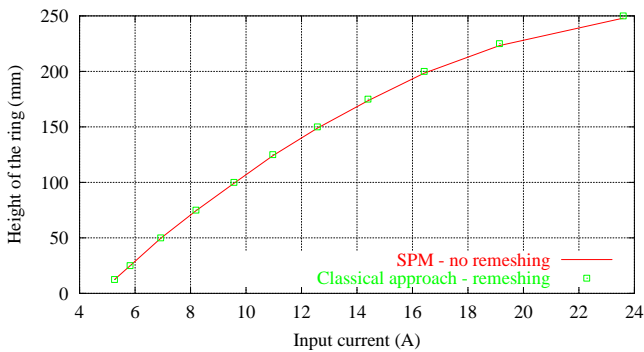


Fig. 1. TEAM problem 17: height of the ring versus input current (50 Hz).

### B. Adding of Changing a Stranded Conductor

For a stranded conductor $\Omega_{s,p}$, the circuit relation relating its current $I_p$ and voltage $U_p$ changes is

$$(\partial_t\,\boldsymbol{a}_p,\boldsymbol{j}')_{\Omega_{s,p}} + (\partial_t\,\boldsymbol{a}_q,\boldsymbol{j}')_{\Omega_{s,p}} + R_p\,I_p = -U_p\,, \quad (7)$$

where $R_p$ is the coil resistance and $\boldsymbol{j}'$ is a global test function defined for the considered coil as $\boldsymbol{j}_{s,p}=N_s/S_s\,\boldsymbol{t}$, with $N_s$ its number of turns, $S_s$ its total surface area and $\boldsymbol{t}$ the unit vector tangent to the coil direction [7]. The contribution from a previous solution $q$, i.e. $(\partial_t\,\boldsymbol{a}_q,\boldsymbol{j}_{s,p})_{\Omega_{s,p}}$, gains to be evaluated indirectly from integrals on the modified regions that were sources of $\boldsymbol{a}_q$; this avoids any integration in $\Omega_{s,p}$ which would need to project $\boldsymbol{a}_q$ on its mesh. For this, in the SP sequence, one gets back to the previous iteration of problem $p$ preceding problem $q$, and uses $\boldsymbol{a}_q$ and $\boldsymbol{a}_p$, respectively, as test functions in their formulations. Subtracting the resulting expressions, only some integrals on the modified regions of problem $q$ remain with the term $(\boldsymbol{j}_{s,p},\boldsymbol{a}_q)_{\Omega_{s,p}}$, which is the time primitive of the term to be evaluated, thus via the other remaining integrals. This is a remarkable result that allows a very accurate calculation of the inductance change, in particular in non-destructive testing problems, as it will be studied in the extended paper.

If no current change is allowed, $I_p$ and $\boldsymbol{a}_p$ are zero in (7), which leads to a voltage change $U_p$ and thus to an inductance change.

For the TEAM problem 17 again, the accuracy obtained with the SPM approach on the calculation of the mutual inductance between the main coil and a search coil is pointed out in Fig. 2. A significant speed-up factor is again obtained with the SPM (about 120) in comparison with the classical approach.
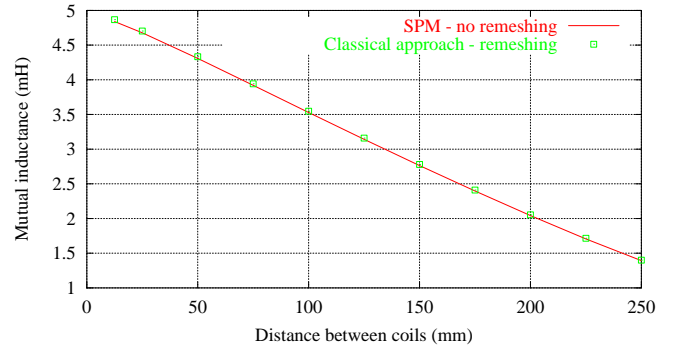


Fig. 2. TEAM problem 17: Mutual inductance as a function of distance between the main coil and a search coil.

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